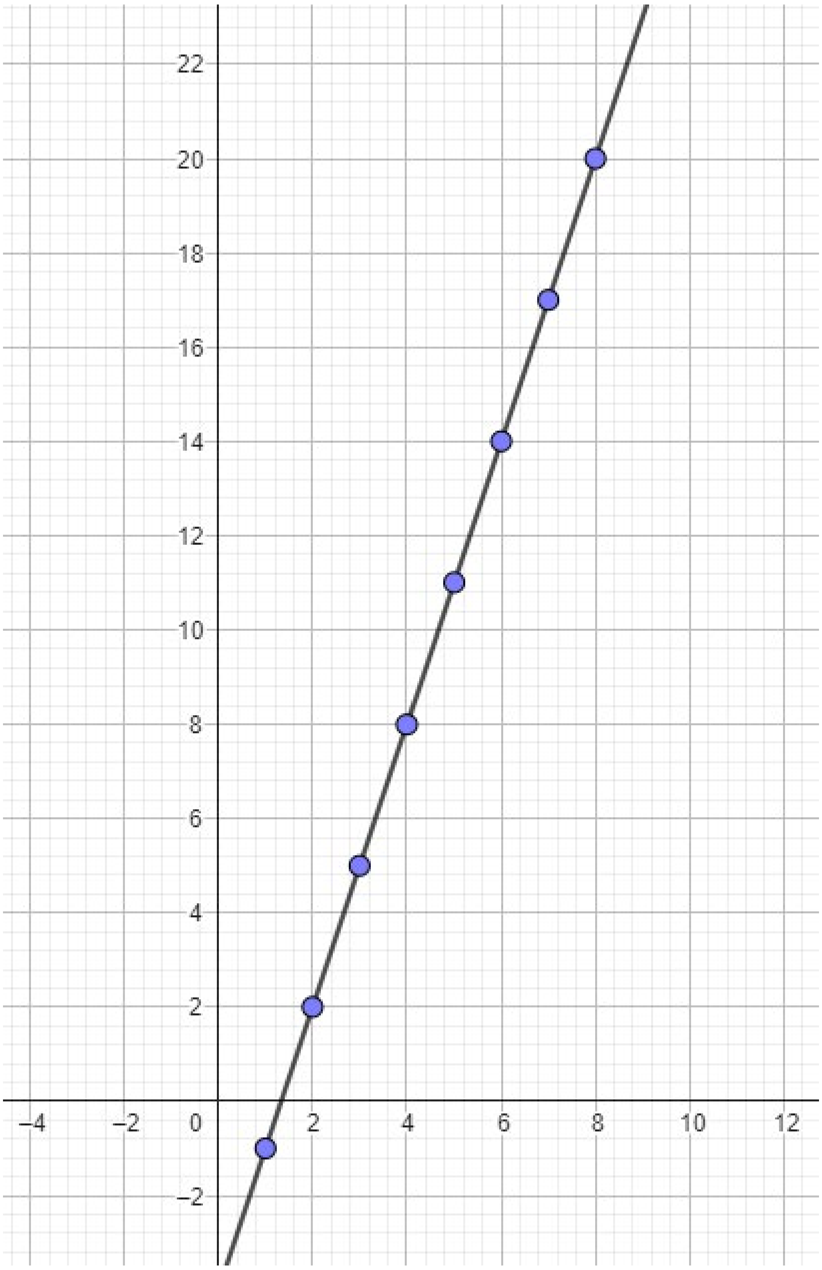
Sequences



On the left, a graph is marked with points along its length.

1 Using the coordinates of each point on the graph, write down the sequence that it represents.

The x coordinate is the term number or n.

The y coordinate is the actual term of the sequence.

2 Use the sequence to derive the nth term.

3 i What would the coordinates of the 200th point on this graph be?

3 ii What would be the 200th term of the sequence?

4 Is this sequence linear? Explain how you know using a sentence about the sequence ***and*** a sentence about the graph.

5 Look at the nth terms below. For each one, state whether the sequence is linear or non-linear. State the first five terms of each sequence.

a $3n^2$

b $2n^3 + n$

c $5n - 7$

d $\frac{n}{4} + 8$

e $9 - n$

f $2n^3 + n^2 - n + 12$

g $23 - \frac{4}{n}$

h $\frac{n!}{2}$

Arithmetic Sequences (Linear Sequences)

6 Fill in the gaps and find the nth term for each of the following sequences.

a 4, 7, 10, 13, 16, ____, ____, …

b 12, 3, -6, -15, -24, ____, ____, ____, …

c 3, ___, ___, ___, 31, ____, …

d 28, ____, ____, ____, ____, ____, ____, ____, ____, -8, …

e ____, ____, 67, 70, ____, ____, ____, …

f ____, ____, 74, ____, ____, ____, 86, ____, …

g 37, ___, ____, ___, ___, 22, ___, ___, …

h ____, ____, ____, ____, 12, ____, -10, ____, …

i ____, ____, 23, ____, 28, ____, ____, …

k ____, ____, ____, 7, ____, ____, 11, ____, …

7 Determine the whether the number given is in the sequence for which the nth term is provided.

a $Is\ 57 \in 7n - 5?$

b $Is\ 182 \in 5n + 2?$

c $Is\ 63 \in 9n - 19?$

d $Is\ 171 \in 8n + n?$

e $Is - 23 \in 17 - 6n?$

f $Is - 13 \in 9 - 4n?$

g $Is\ 837 \in 3n + 113?$

h $Is\ 94 \in 7n + 3?$

If we have an arithmetic sequence, another way of determining the nth term is the substitute the numbers into the following formula.

$$nth\ term = a + (n-1)d$$

Here, ***a is the first term*** of the sequence and **d is the common difference**.

Worked Example (COPY THIS INTO YOUR BOOK AND HIGHLIGHT THE BORDERS)

$$nth\ term = a + (n-1)d$$

The second term in a sequence is 66. The 29th term in the sequence is 147. What is the 500th term in the sequence?

$$d = \frac{147 - 66}{29 - 2} = \frac{81}{27} = 3$$

$$nth\ term = (66 - 3) + 3(500 - 1)$$

$$= 63 + 1497$$

$$= 1560$$

8 Work out the common difference and the first term and use this information to find the unknown term in each of these arithmetic sequences.

a. The first term of a sequence is 34. The common difference is 6. Find the 500th term of the sequence.
b. The second term of a sequence is 98. The common difference is 5. Find the 70th term of the sequence.
c. The first term of a sequence is 84. The second term is 40. Find the 405th term of the sequence.
d. The fourth term of a sequence is 60. The fifth term of the sequence is 67. Find the 850th term of the sequence.
e. The ninth term of a sequence is 65. The twelfth term is 98. Find the 450th term of the sequence.
f. The seventh term of a sequence is 18. The fifteenth term is 202. Find the 303rd term of the sequence.
g. The twenty-seventh term of a sequence is 12. The twentieth term in the sequence is 68. Find the 208th term of the sequence.

Quadratic Sequences

9 Use the method of finding differences and then finding differences again to determine the nth term for the following equations.

a 56, 44, 36, 32, 36

b -61, -56, -45, -28, -5, 24

c 31, 67, 119, 187, 271, 371

Fibonacci Sequences

Leonardo Fibonacci first published details of this sequence.

The general formula is:

$$F_n = F_{n-1} + F_{n-2}$$

This means that you get the next number by adding the previous two numbers together.

ie 1, 1, 2, 3, 5, 8, …

10 If the first two numbers of a Fibonacci sequence are ***a*** and ***b***, calculate an expression for each of the numbers up to F_{10} .

11 Investigate the ratio of $\frac{F_n}{F_{n-1}}$. You can use a calculator for this question.

Polygonal Sequences

Polygonal sequences include things like triangular numbers, square numbers, cubic numbers, pentagonal numbers etc.

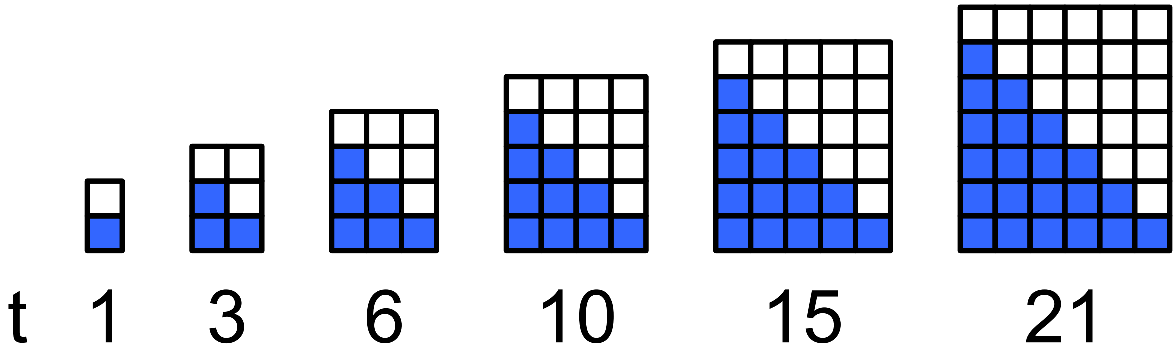
We are going to concentrate on the triangular, square and cubic numbers.

Triangular Numbers are great because they allow you to work out what the sum of all the numbers up to a number is.

12 Complete the following sequence:

1, 3, 6, 10, 15, ___, ___, ___, ___, ___, ___, ___, …

13 Look at the pattern below. Can you form an equation to describe the pattern.



Carl Gauss came up with the formula to allow you to find the sum of all numbers up to 100 when he was 8 years old. He then went on to solve many more difficult problems in mathematics.

14 What is the sum of all the numbers up to 100?

15 What is the sum of all the numbers up to 5000?

16 Write down all the square numbers from 1^2 to 20^2. Examine the differences between consecutive square numbers. Is there a pattern there?

17 Write down all the cube numbers between 1^3 and 10^3.

18 Use your calculator to find the value of $\frac{(n+1)^3}{n^3}$ for each of these cube numbers up to 10^3. Can you find a pattern?